A New Co-located MIMO Radar System for Multi-target Tracking and Localization

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Abstract – *Multiple Input Multiple Output (MIMO) radars are a new generation of radar systems that bring with them many benefits compared to traditional phased-array and multistatic radars. Target localization using MIMO radars with co-located antennas has been recently discussed in the literature. It has been shown that the maximum number of targets that can be uniquely localized in one cell is bounded. This paper presents a new application of MIMO radars in Multi-Target Tracking (MTT) problems. Firstly, the previous model for co-located MIMO radars is modified in order to guarantee the observability in received measurements. Afterwards, it is shown that using prior information about the motion of targets relaxes the limitation on the number of uniquely localized targets. For filtering part, an Unscented Kalman Filter (UKF) algorithm is used to update states of targets. Simulation results confirm the superiority of proposed approach in estimating states of multi-targets falling in the same resolution cell.*

Keywords: MIMO radars, uniquely detectable targets, multi-target tracking, localization.

1 Introduction

Multiple Input Multiple Output (MIMO) radars are a new generation of radar systems that bring with them many benefits compared to traditional phased-array and multistatic radars. The advantages of MIMO radars have urged researchers to explore the capability of MIMO radar systems in different contexts such as target detection, target localization and tracking, waveform design and antenna allocation.

Detection capability of MIMO radars has been vastly explored in the literature [1][2][3][4][5][6]. The superiority of MIMO radars to traditional phased-array and multistatic radars in target detection has been discussed in [2] and [7], respectively. It is also noted that sending orthogonal signals provide some benefits for co-located MIMO radars. Beside better detection capability, it has been demonstrated in [4][2] that the number of targets that can be uniquely detected by a MIMO radar is more than that of targets detected by phased-array radars. Also, Cramer-Rao Lower Bound (CRLB) of target estimation for MIMO radars has been derived in [2]. Results show that the orthogonality of transmitted signals provides lower CRLB than a radar system with coherent transmitted signals.

Target localization techniques have been well-developed for co-located MIMO radars. A maximum-likelihood based technique for Direction-of-Arrival (DOA) estimation using co-located MIMO radars has been proposed in [4][8]. A good review of different techniques for target localization with several targets within one resolution cell has been also presented in [9]. However, none of the above methods can provide an estimate of range. In other words, the effect of each target has been only considered in the cell in which the target is located. Therefore, received measurements do not have any information about range. This problem may damage the observability of measurement model, especially, in tracking contexts because targets with the same angle and different ranges provide the same measurement until they are in the same cell.

Parameter identifiability is the other common problem in target localization. The maximum number of uniquely detectable targets for phased-array radars with mono-pulse processing has been found in [10]. The same has been derived by [11][8] for MIMO radars where the maximum number of uniquely estimated targets varies between $[M + N - 1, M \times N - 1]$ with M and N as the number of transmitters and receivers, respectively. The main reason for the change in the maximum bound is the geometry of antennas in the array of transmitters and receivers providing different numbers of unique elements in the system matrix. Although the proposed bounds have been found based on the previous model (considering the effect of targets only in one cell), algebraic manipulations in this paper will find the same bound even for the new model. Therefore, localization techniques may fail in providing accurate estimates of targets when the number of targets in one cell exceeds the given bound.

A tracking algorithm has been proposed by [12] in order to relax the maximum bound of target localization derived in [10][13]. The tracking method has been also applied to co-located MIMO radars by [14]. The main idea is to bring prior information of targets inside estimation in order to estimate more number of targets than the given bound. Also, [12][14] have shown that not only can the new tracking method relax the maximum bound on target estimation, but also it facilitates estimating states of closely spaced targets. However, the approach given in [14] suffers from the non-observability of signal model and, consequently, there is no guarantee that tracking results become satisfactory for all scenarios.

In this paper the previous model used for MIMO radars in the literature is first modified. Then, the localization technique is developed for the new model in order to estimate DOA and range of every target. The maximum number of uniquely localized targets is also found based on the new model. The main contribution is to propose a tracking procedure so that more number of targets than the given bound is estimated in one cell. To do this, samples are generated around the predicted location of targets based on the estimated covariance. An Unscented Kalman filter (UKF) [15] algorithm is, then, used in order to update states of targets. The main idea is to distribute sigma points in different cells so that the uncertainty of estimation is taken into account.

The rest of paper is organized as follows. Section 2 describes the co-located MIMO radar, signal model, and different structures. Target localization and maximum bound on the number of targets are discussed in section 3. Tracking algorithm is also presented in section 4. Simulation results comparing tracking and localization techniques are given in Section 5. Finally, the paper will be concluded in section 6.

2 Co-located MIMO Radars

It is assumed in this paper that arrays of transmitters and receivers are co-located, which means the inter-elemental distance between each two antennas is much smaller than the distance to targets. Also, origins of arrays are so close that resolution cells are characterized by a group of circles. The separated-array case with elliptical resolution cells has been tackled in [8]. The following assumptions are made for the MIMO radar and target models used in this paper:

- Without loss of generality, the centroid of antennas is assumed to be origin ($[0\ 0]^T$).
- The number of targets initially present at each cell is less than maximum bound found later. However, the number can exceed the bound in next steps.
- The number of targets in the scenario is fixed. An algorithm is being developed in order to tackle time varying number of targets.
- Objects are point sources and modeled as Swerling type I [16] targets. That is, target scatters change every scan but remain constant during each scan.

Given signals in L snapshots, input signals are orthogonal meaning that the covariance matrix of inputs is diagonal. The covariance matrix R is defined as

$$R = \sum_{l=1}^{L} \mathbf{s}(l)\mathbf{s}^*(l) \tag{1}$$

where $\mathbf{s}$ is the transmitted signal that is an $M \times 1$ vector. Assuming T targets in an arbitrary cell, the reflected signal of the t-th target can be written in the following form [4]:

$$\mathbf{z}(l) = \alpha_t A_t \mathbf{s}(l) \tag{2}$$

Here, $\mathbf{z}(l)$ is the reflected signal in the l-th snapshot, A_t is the system matrix, and α_t is the random scatter of the t-th target. Random scatter is a complex number whose real and imaginary parts are both Gaussian distributed with mean and variance $\{\hat{\alpha}_t^R, \hat{\alpha}_t^I\}, \sigma_\alpha^2$, respectively. The system matrix can be found based on the co-located antennas and far enough targets as [4]

$$\begin{aligned}
A_t &= a_t^r a_t^t \\
(a_t^t)_i &= \exp\left(-j\frac{2\pi}{\lambda}\left(\sin(\theta_t)x_{ti} + \cos(\theta_t)y_{ti}\right)\right) \\
(a_t^r)_l &= \exp\left(-j\frac{2\pi}{\lambda}\left(\sin(\theta_t)x_{rl} + \cos(\theta_t)y_{rl}\right)\right) \\
i &= 1, ..., M, l = 1, ..., N
\end{aligned} \tag{3}$$

where λ is the wavelength, M and N are the number of transmitters and receivers, respectively, a_t^t and a_t^r are transmitter and receiver steering vectors, respectively, θ_t is target DOA, and $[x_{ti}, y_{ti}]$ and $[x_{rl}, y_{rl}]$ denote the location of the i-th transmitter and l-th receiver, respectively. Now, define r_t as the t-th target range with regard to the origin. Each target affects two neighbor cells depending on its range. Assume the t-th target is located in the c-th cell. The contribution of the t-th target can be observed in the sampled signal in both cells c and $c-1$. Defining β_t as the ratio of original target's reflection observed in the c-th cell, the contribution of the t-th target in the c-th and $c-1$th cell can be written as [11][13]

$$\begin{aligned}
\mathbf{z}_t^c &= \beta_t \mathbf{z}_t \\
\mathbf{z}_t^{c-1} &= (1-\beta_t)\mathbf{z}_t
\end{aligned} \tag{4}$$

where β_t is defined as

$$\beta_t = \frac{r_t + r_{bin} - r_c}{r_{bin}} \tag{5}$$

with r_c as the radios of the c-th cell. Now, suppose that there are T_1 and T_2 targets in the c-th and $c+1$-th cells, respectively. The received signal in the c-th cell is written as

$$\mathbf{z}^c = \sum_{t=1}^{T_1} \beta_t \mathbf{z}_t + \sum_{t'=1}^{T_2} (1-\beta_{t'})\mathbf{z}_{t'} + \mathbf{w} \tag{6}$$

where the index l has been removed for simplicity. Also, $\mathbf{w}$ denotes complex additive noise that is Gaussian distributed with zero mean and variance σ_w^2. In practice, output of matched filters are usually used instead of original signals. The matched filter output is written as

$$E^c = \frac{1}{\sqrt{L}} \sum_{l=1}^{L} \mathbf{z}^c(l)\mathbf{s}^H(l) \tag{7}$$

where $\mathbf{s}^H$ denotes complex transpose of vector $\mathbf{s}$. Defining $\eta^c = \mathrm{VEC}(E^c)$, it is straightforward to show that the output of matched filter is also a summation of the contribution of each target to the corresponding cell as [10][13]

$$\eta^c = \sum_{t=1}^{T_1} \beta_t \eta_t + \sum_{t'=1}^{T_2} (1-\beta_{t'})\eta_{t'} + \mathbf{w}' \tag{8}$$

Here, η_t are the matched filter outputs when only the t-th target is available, and $\mathbf{w}'$ is the additive noise that is Gaussian with zero mean and variance σ_w^2.

3 Target Localization and Parameter Identifiability

The following nonlinear model can be written for the η_t described in the last part [4]

$$\eta_t = \alpha_t \mathbf{d}_t \tag{9}$$

with the following definition for the new system matrix

$$\mathbf{d}_t = \sqrt{L}\mathrm{VEC}\left(A_t U \Gamma^{\frac{1}{2}}\right) \tag{10}$$

where U, Γ are Singular Value Decomposition (SVD) of covariance matrix R, and $\mathrm{VEC}(A)$ denotes the vectorized form of matrix A. It is shown that output of matched filter is sufficient statistics for detection and localization [10][13]. Newman-Pearson test for detection provides the following inequality for each cell [2][4]:

$$\omega = \begin{cases} H_1 & \text{If} |\eta^c| \geq \gamma \\ H_0 & \text{Otherwise} \end{cases} \tag{11}$$

Here, H_1, H_0 are hypotheses corresponding to the presence and absence of targets in the cell, respectively. Also, γ is the threshold determined according to the probability of false alarm. Now, the set of detected cells is defined as $\Phi = [\eta_1, ..., \eta_C]$ where C is the number of detected cells. A Maximum Likelihood (ML) estimator is used to estimate parameters of targets where the parameter vector of each target is defined as

$$\Theta_t = [\theta_t, \beta_t, \hat{\alpha}_t^R, \hat{\alpha}_t^I] \tag{12}$$

Here, it is assumed that all variances are known, however, it does not disturb the generality of the algorithm. Detected cells are classified to different clusters based on the distribution of targets in the surveillance region. Each cluster may be composed of several cells occupied by a group of targets. As an example, consider a scenario with 3 targets. Figure 1 shows some possible distribution of targets in 5 neighbor cells. The up-right scenario provides all three targets in C_2. Therefore, the cluster involves $\{C_1, C_2\}$. However, in the down-right graph, there are two clusters involving $\{C_1, C_2, C_3\}$ and $\{C_4, C_5\}$. Consequently, distribution of targets determines the number of clusters as well as the number of cells in each cluster.

Without loss of generality, it is assumed that there is only one cluster. The likelihood function is written as $p(\Phi|\Theta_1, ..., \Theta_T) = \mathcal{N}(\mu, \Sigma)$ where $\mathcal{N}$ denotes a normal distribution with mean μ and covariance Σ defined by (13).

Individual means are found as

$$\mu_i = \sum_{t=1}^{T_i} \beta_t \hat{\eta}_t + \sum_{t'=1}^{T_{i+1}} (1-\beta_{t'})\hat{\eta}_{t'} \tag{14}$$

with $\hat{\eta}_t = \hat{\alpha}_t \mathbf{d}_t$. The same expressions can be found for the covariance knowing that $E\{(\alpha_t - \hat{\alpha}_t)(\alpha_t - \hat{\alpha}_t)'\} = 2\sigma_\alpha^2$. Detailed expressions of covariance matrix are not given here due to space limitation but the similar equations can be found in [10][13]. The negative log-likelihood function can be now defined as

$$\Gamma = \frac{1}{2}\log|\Sigma| + \frac{1}{2}(\Phi - \mu)\Sigma^{-1}(\Phi - \mu)' \tag{15}$$

The goal is to find the set of parameters Θ_t and the number of targets at each cell, T_i, in order to minimize the above cost function. To do so, a Minimum Distance Length (MDL) criterion is used to find the correct number of targets at each cell. Details of MDL can be found in [10][13] but the basic idea behind MDL method is to construct different hypotheses based on an arbitrary number of targets in different cells. Given T_{max} as the maximum number of targets in the surveillance region, $T^h = \{T_2, ..., T_C\}$ represents the h-th hypothesis where $T_i \in \{0, ..., T_{max}\}$ [1]. MDL evaluates the cost of every hypothesis and, then, the best hypothesis gives the true number of targets as well as their estimates.

[1]Note that the first cell does not include any target because the signal is due to the contribution of targets in the neighbor cell.

$$\mu = [\mu_1, ..., \mu_C], \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & 0 & 0 & 0 & 0 \\ \Sigma_{21} & \Sigma_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & . & . & . & . \\ . & . & . & . & \Sigma_{(C-1)(C-1)} & \Sigma_{(C-1)C} \\ 0 & 0 & 0 & 0 & \Sigma_{C(C-1)} & \Sigma_{CC} \end{pmatrix} \quad (13)$$

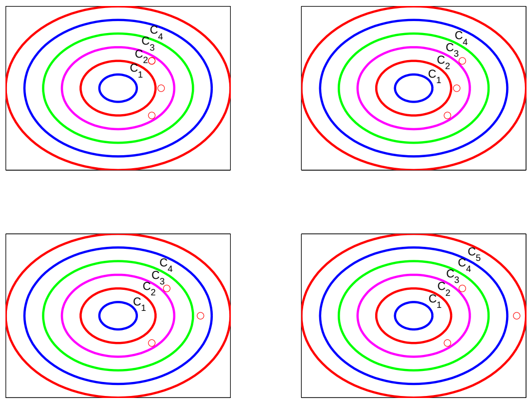

Figure 1: Different clusters of cells based on distribution of targets.

3.1 Parameter Identifiability

Parameter identifiability has been discussed for MIMO radars in [11]. It is shown that the maximum number of targets that can be uniquely localized belongs to the set $[M+N-1, M\times N-1]$. The main idea behind the given bound is to count the maximum unique elements of system matrix A_t. Depending on the distribution of antennas, the maximum number of uniquely detectable targets changes in above given interval. However, the derived bound is based on the previous model in which the contribution of a target is only considered in its own cell. The question is whether the bound changes when the model in this paper is taken into consideration. It can be inferred from (15) that Γ is a function of elements of Σ and η. In other words, cost function can be written as $\Gamma = \Gamma_1\left(\Sigma_{11}, ..., \Sigma_{(MNC)(MNC)}\right) + \Gamma_2\left(\mu_1, ..., \mu_{MNC}\right) + \Gamma_3\left(\Sigma_{11}, ..., \Sigma_{(MNC)(MNC)}, \mu_1, ..., \mu_{MNC}\right)$ where Γ_is can be found after expanding the original cost function and writing the function in terms of Σ and μ, separately. Without loss of generality, assume that $C = 1$ which means that all targets are located in one cell. The necessary condition for the uniqueness of optimal parameter is the full rankness of jacobian matrix [10][13]. Jacobian is found with regard to covariance and mean because they are direct functions of parameters. Defining $g_1 = [\Sigma_{11}, ..., \Sigma_{(MN)(MN)}]$ as the vectorized form of covariance, the following equations are derived for jacobian:

$$G_1 = \begin{pmatrix} \frac{\partial g_1}{\partial \Theta_1^i} & . \; . \; . & \frac{\partial g_1}{\partial \Theta_T^i} \\ . & . \; . \; . & . \\ . & . \; . \; . & . \\ . & . \; . \; . & . \\ \frac{\partial g_{(MN)^2}}{\partial \Theta_1^i} & . \; . \; . & \frac{\partial g_{(MN)^2}}{\partial \Theta_T^i} \end{pmatrix}$$

$$G_2 = \begin{pmatrix} \frac{\partial \mu_1}{\partial \Theta_1^i} & . \; . \; . & \frac{\partial \mu_1}{\partial \Theta_T^i} \\ . & . \; . \; . & . \\ . & . \; . \; . & . \\ . & . \; . \; . & . \\ \frac{\partial \mu_{MN}}{\partial \Theta_1^i} & . \; . \; . & \frac{\partial \mu_{MN}}{\partial \Theta_T^i} \end{pmatrix} \quad (16)$$

where Θ_t^i denotes the i-th parameter of the t-th target. If the necessary condition is not satisfied for none of the above matrices, there will be at least two parameters Θ_1 and Θ_2 for which $G_i\Theta_1 = G_i\Theta_2$. In this case, the linear approximation of cost function gives the same optimal value for two sets of parameters arguing the uniqueness. Covariance matrix does not include any information about target scatters because it is a function of σ_α^2. Therefore, columns of G_1 become zero when the parameter of interest is target scatter. In addition, covariance matrix does not give any information about DOAs. This indicates that G_1 cannot provide any unique solution because it is not full-rank. Therefore, uniqueness of solutions should be justified by exploring G_2 that is a function of all parameters. It can be easily shown that G_2 is a function of system matrix $\mathbf{d}$. Referring to (10) the maximum number of unique elements in the system matrix is its length $M \times N$. Therefore, vector μ has maximum $M \times N$ unique elements that is, indeed, its length. This leads to a conclusion that the maximum number of unique columns in G_2 cannot exceed the given bound. Consequently, the number of parameters T should be less than or equal to MN. The same analysis can be done for other parameters of every target. It is also possible to make the same conclusion for targets lying in multiple cells $C > 1$. In this case, the necessary condition is found for each cell separately.

One important point should be made here about the bounds found above. The enlightened bounds are the only necessary conditions for the uniqueness of solutions. Geometry of antennas and targets in the surveillance region are very important in the number of accurate estimates of targets. For example, although the maximum number of unique elements in $\mathbf{d}$ is $M \times N$, the bound becomes smaller if antennas are distributed along with the x-axis. In this case, [11] have found the range $[M+N-1, MN-1]$ as the maximum bound when the distance of antennas determines the actual bound. Therefore, the above discussions provide some intuitions about the maximum possible number of targets that can be uniquely localized in one cell.

4 Tracking Algorithm

The maximum bound in target localization can be troublesome in practice. Targets may enter one resolution cell in some time steps and, therefore, get unresolved for the radar system. Also, even when the number is less than the bound, localization algorithm may not provide good results due to the geometry of targets. To handle the limitation on the number of targets prior information of targets' movement is used. Assume the following common near constant velocity model for the t-th target:

$$\mathbf{x}_t(k+1) = F_t \mathbf{x}_t(k) + G_t \mathbf{v}_t(k) \hat{\alpha}_t(k+1) = \hat{\alpha}_t(k) + \delta_\alpha$$

where k is time step, $\mathbf{v}_t(k)$ is the additive noise that is Gaussian distributed with zero mean and covariance matrix Q_t, F_t, G_t are motion matrices, and δ_α is the Gaussian additive noise modeling the uncertainty in target scatters. Motion matrices can be defined as

$$F_t = \begin{bmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_s \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

$$G_t = \begin{bmatrix} \frac{T_s^2}{2} & 0 \\ T_s & 0 \\ 0 & \frac{T_s^2}{2} \\ 0 & T_s \end{bmatrix} \quad (18)$$

with T_s as the sampling time. In this paper, the number of targets in the surveillance region is assumed to be time invariant. Dealing with time variant number of targets happening due to new targets' birth or available targets' death is difficult when MIMO radars are used as measurement models. In this case, the number of targets varies in each cell by time and some more advanced techniques should be developed in order to find the number of targets and individual estimates. This topic will be discussed in future works. Given T targets and their estimates $\mathbf{x}_t(k|k)$ at the k-th step time, a Bayesian estimator finds the posterior probability distribution of targets given measurements. Measurements are outputs of matched filters defined as $\Phi(k)$ in the previous part. The key issues in multi-target tracking using the underlying MIMO radar are

- Grouping
- Data Association
- Hypothesis Generation
- Likelihood Computation
- Estimates Update

The first step is grouping which is a simple classification of different targets based on their location. Group target tracking has been extensively discussed in the literature [17]. Here, targets are classified to different groups based on the associated cell. Figure 1 shows how different groups of targets may be constituted. The same grouping technique is done here at each time step. For simplicity, it is assumed that there is one group (cluster) at the k-th step time. In scenarios with more than one group, the superscript g is used for all parameters to clarify the category of targets and measurements.

4.1 Data Association

Let's define $\mathcal{T}_t(k)$ as the t-th track at the k-th step time. Each track is represented by its "sufficient statistics" summarizing the information content of measurements [18]. As an example, the t-th track can be represented for an Extended Kalman Filter (EKF) estimator as

$$\mathcal{T}_t(k) = \{\mathbf{x}_t(k|k), P_t(k)\} \quad (19)$$

where $\mathbf{x}_t(k|k), P_t(k)$ are estimated mean and covariance for the t-th track, respectively. The set of all tracks at the k-th time step is written as

$$\mathcal{T}(k) = \left\{\mathcal{T}_1(k), ..., \mathcal{T}_{N(k)}(k)\right\} \quad (20)$$

where $N(k)$ is the number of confirmed tracks at the k-th time step. A cell-to-track association matrix is defined as

$$\gamma_{tc} = \begin{cases} 1 & \text{If the } t\text{-th track is associated with the } f\text{-th cell} \\ 0 & \text{Otherwise} \end{cases} \quad (21)$$

The same matrix can be defined for track-to-cell association ξ_{ct}. Unlike common tracking context in which each measurement is only associated with one track, here, there is no such condition over rows of association matrix. However, targets are small enough to fit inside a cell and, therefore, the following condition is yet valid

$$0 \leq \sum_{c=1}^{C} \gamma_{tf} \leq 1, t = 1, ..., N(k) \quad (22)$$

The t-th target is assigned to the c-th cell if the following inequality is satisfied

$$\frac{(r_t(k|k-1) - r^c)^2}{\sigma_r^2} \leq \zeta \quad (23)$$

where $r_t(k|k-1)$ is the predicted range of the t-th target, r^c is the range of the c-th cell, and ζ is the threshold. Threshold is found so that the cell falls in 95 percent of confidence region. A target may be assigned to two cells if it is located in the neighbor of assigned cells. In above equation, σ_r^2 represents the variance of range measurements. The variance of range estimation can be found by linearizing the range model and, then, using the covariance of estimation $P_t(k)$.

4.2 Hypothesis Generation/Evaluation

The h-th hypothesis ($\mathcal{H}_h$) is indeed a target-to-cell association for a group of targets located in the underlying cluster. As targets may be assigned to more than one cell, different hypotheses are formed for a multi-target scenario. To evaluate each hypothesis, its likelihood is found and the one with the most value is taken as the winner. The Likelihood function for the h-th hypothesis can be written as

$$p(\Phi|\mathcal{H}_h) = p(\Phi|\gamma^h, \mathcal{T}^h(k)) \tag{24}$$

where $\mathcal{T}^h(k)$ are those targets lying in the same cluster based on the h-th hypothesis. Note that each association involves information about the location of targets. According to Figure 1, targets may be fallen in different clusters because, in the new model, the effect of targets in the neighbor cell is also taken. Therefore, a target belonging to one cluster at the k-th time step may be assigned to a separate cluster in the next step. The above likelihood can be easily found using the expressions give in (13) and (15). The hypothesis with the lowest negative likelihood is chosen as the winner and its estimates are considered as new tracks.

4.3 Estimates Update

States of targets are updated for each hypothesis separately. The update step is to find a new set of tracks $\mathcal{T}^h(k+1)$ from the previous step tracks $\mathcal{T}^h(k)$, the h-th association and measurements at the $k+1$-th time step. Due to nonlinearity of measurement model, traditional Kalman Filter or EKF [19] provide poor results. In this paper, an Unscented Kalman Filter (UKF) method is used for states update. The UKF algorithm first proposed by [15] has been successfully applied to many nonlinear filtering problems [20]. Due to the dimension of states that may be considerably large when multiple targets fall in different cells, UKF method is the best choice because other nonlinear filtering methods such as particle filtering [21] suffer from the curse of dimensionality. Let's define $\mathcal{T}_t(k+1|k)$ as the predicted state for the t-th available target with the same that can be found for the prediction of target scatters $\hat{\alpha}_t(k+1|k)$. Predicted DOA and range for each target can be computed as

$$\begin{aligned} \theta_t(k+1|k) &= \tan^{-1}\left(\frac{y_t(k+1|k)}{x_t(k+1|k)}\right) \\ r_t(k+1|k) &= \sqrt{(x_t(k+1|k))^2 + (y_t(k+1|k))^2} \end{aligned} \tag{25}$$

The predicted range coefficients $\beta_t(k+1|k)$ can be easily found with the above predicted range and (5).

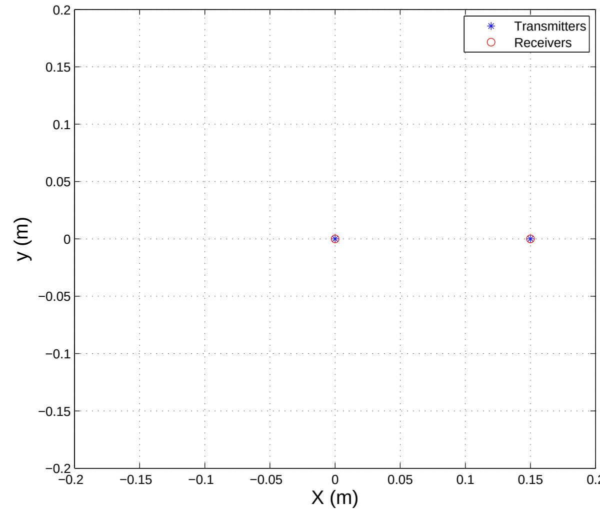

Figure 2: Distribution of transmitters and receivers in the surveillance region.

Therefore, predicted parameter vector can be written as $\Theta_t(k+1|k) = [\theta_t(k+1|k), \beta_t(k+1|k), \hat{\alpha}_t^r(k+1|k), \hat{\alpha}_t^I(k+1|k)]$ and the parameter vector for all targets is indicated by $\Theta(k+1|k)$. Let's define $\Theta^h(k+1|k) \subset \Theta(k+1|k)$ as those targets falling in the same cluster based on the h-th hypothesis. A UKF algorithm gets predicted states and scatters $\{\mathcal{T}^h(k+1|k), \hat{\alpha}^{R,h}(k+1|k), \hat{\alpha}^{I,h}(k+1|k)\}$ (or equivalently parameter vector), corresponding covariances, and measurements $\Phi(k+1)$ in order to provide a new set of states and covariances $\{\mathcal{T}^h(k+1), P^h(k+1)\}$ and target scatters and corresponding variance. Different steps of UKF method are ignored here but details can be found in [15]. New estimated tracks and corresponding covariances are used to evaluate the likelihood of each hypothesis discussed in the last part. Finally, the winner hypothesis includes information of final states.

5 Simulation Results

A MIMO radar with 2 transmitters and receivers is considered for simulation purposes. Figure 2 shows the distribution of antennas in the surveillance region. Other parameters of the radar system are listed in Table 1. First, a single target scenario is considered where initial parameters of the target are $\{r = 820, \theta = \frac{\pi}{3}, \alpha = 3\}$. Signal-to-Noise Ratio (SNR) level is defined as $\text{SNR} = \frac{MN|\alpha|^2}{\sigma_w^2}$. Now, tracking algorithm proposed in this paper is applied to the designed scenario. For comparison, localization algorithm is applied to received signals in every scan and, then, estimated locations of targets are used as measurements for a Kalman filter used for tracking. Figure 3 shows the estimated Root Mean Square Error (RMSE) of estimation after 100 Monte Carlo run. It can be ob-

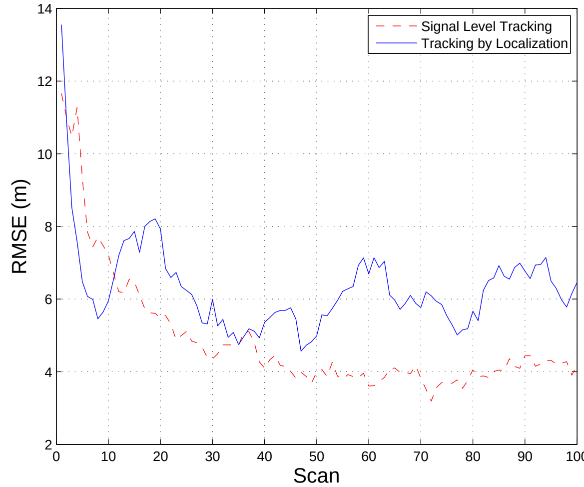

Figure 3: Signal and measurement level tracking RMSE for a single target scenario.

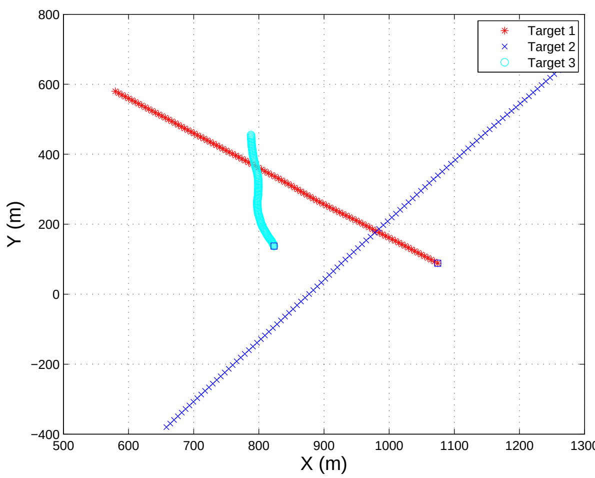

Figure 4: The generated 3-target scenario.

served that signal-level tracking has yield better RMSE bound compared to measurement-level tracking. Also, RMSE curve of measurement-level tracking shows some fluctuations while that of signal-level tracking is much smoother. The same fluctuation can be observed in CRLB of localization showing that measurement-level tracking inherits these fluctuations from inaccuracy in localization. In the second experiment, a 3-target scenario presented in Figure 4 is taken into account. Targets are initially in different cells but they move towards one cell, enter the same cell and stay within the cell for a period of time and, then, leave the cell. Although the maximum bound on the number of targets is 3, localization results are not satisfactory when all targets occupy the same cell. Also, two targets are very closely-spaced in the cell making localization results much worse. Re-

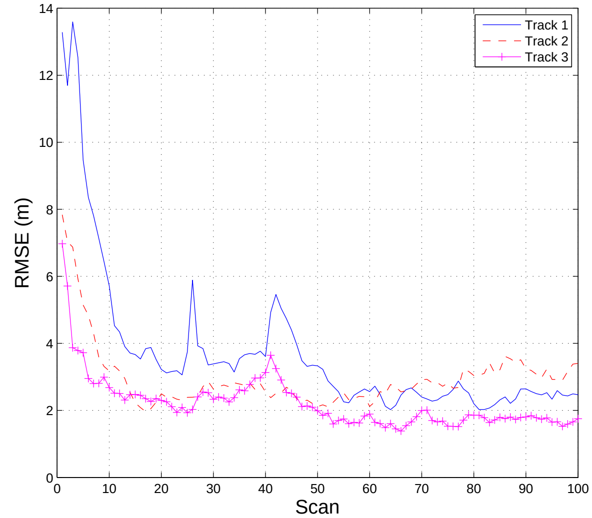

Figure 5: RMSE of estimation using signal level tracking approach.

sults of tracking algorithm are shown in Figure 5 for 100 Monte Carlo run. It can be observed that RMSE converges after some step times but increases when targets enter the same cell and approach each other. However, tracker catches target even in the aforementioned period of time. This Figure shows the capability of algorithm in tracking multiple targets falling within one cell. The same simulation can be done for scenarios with more targets in one cell.

6 Conclusions

A co-located MIMO radar system with a new signal model considering the effect of targets in neighbor cells was discussed in this paper. The localization algorithm was developed for the new model to estimate DOA, range and target scatter. The maximum bound on the number of targets that can be uniquely detected in one cell was derived. Then, a signal-level tracking was proposed in order to relax the bound on the number of estimated tracks. Simulation results show the superiority of signal-level tracking to the measurement-level tracking in a single target tracking case. Also, the performance of tracking was evaluated for scenarios with multiple targets in one cell. It was demonstrated by simulations that the new signal-level tracking method works well for tracking multiple targets lying in the same cell even when the number approaches the maximum bound.

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